

A Study of Rotating Principle Stresses Influence

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ECS Simulation Conference 2023
May 10-11, Linz, Austria



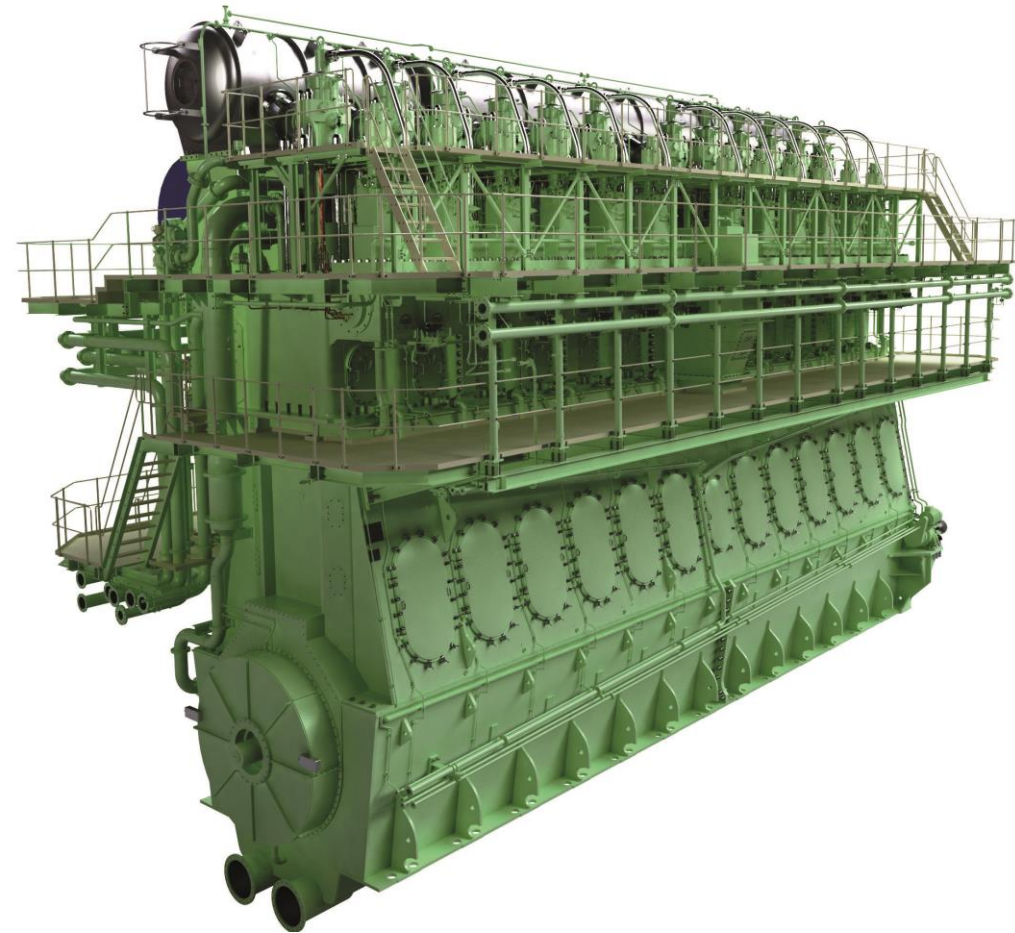
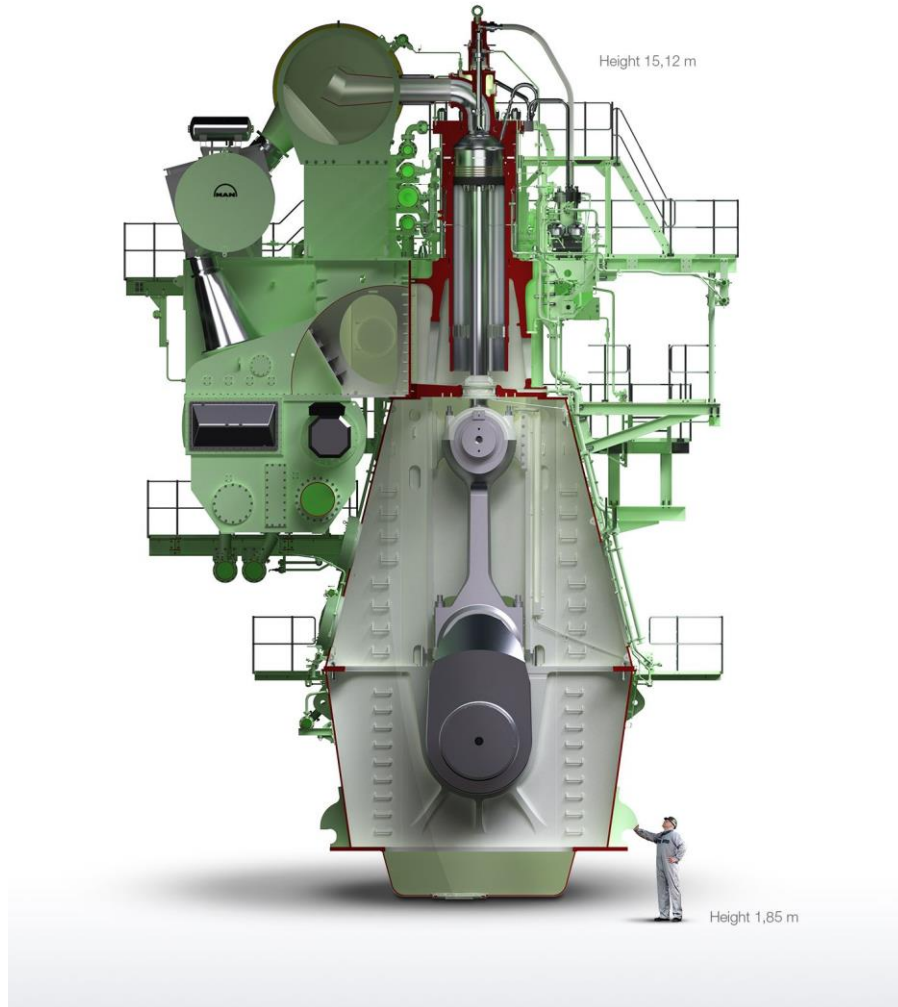
Products

Marine Two-Stroke Engines

Power: 2.000 - 82.000 kW

Cylinder Bore: 30 - 95 cm

Stroke: 1.3 - 3.7 m



Crankshaft

Component with Multiaxial Stresses



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Introduction

Rotating Principle Stresses Influence

- Reduces the fatigue limit
- Empirical penalty factor (S_{NP})
- Fitted to steel fatigue tests
- Only relevant for some material types

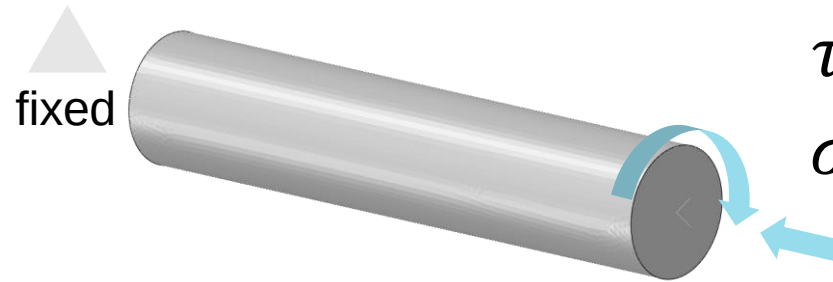


General Factors	Surface Treatment	WELD	SPOT
Stress Gradient			
Gradient Computation Method			FEMFAT 2.4
☒ Endurance Limit ☒ Slope / Cycle Limit			FEMFAT 2.4
Mean Stress			
☒ Endurance Limit			FEMFAT 4.1
☒ Slope / Cycle Limit			FEMFAT 5.1
☒ Surface Roughness			FKM / IABG (Rz)
☐ Constant Stresses			FEMFAT 2.0
☐ Mean (and Amplitude) Stress Rearrangement			PLAST Mean: Without Sequence Influ
☒ Modified Haigh Diagram (Ultimate Tensile Strength)			Stress Gradient Influence
☐ Technological Size Influence			FKM-Guideline
☐ Statistical Influence			Gauss (LogN)
☐ Isothermal Temperature Influence			FEMFAT 4.6
☐ Cast Microstructure			Method of Variable Slopes
☐ Effective Plastic Strain			
☐ Tempering Influence (for Tempering Steel only)			
☐ Surface Residual Stresses			
☐ Boundary Layer			
☐ Fiber Orientation			Logarithmic interpolation
☐ Local Material Properties			
☒ Rotating Principal Stresses Influence			FEMFAT 4.2
Combination Method Influence Factors			FEMFAT 5.1

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Background

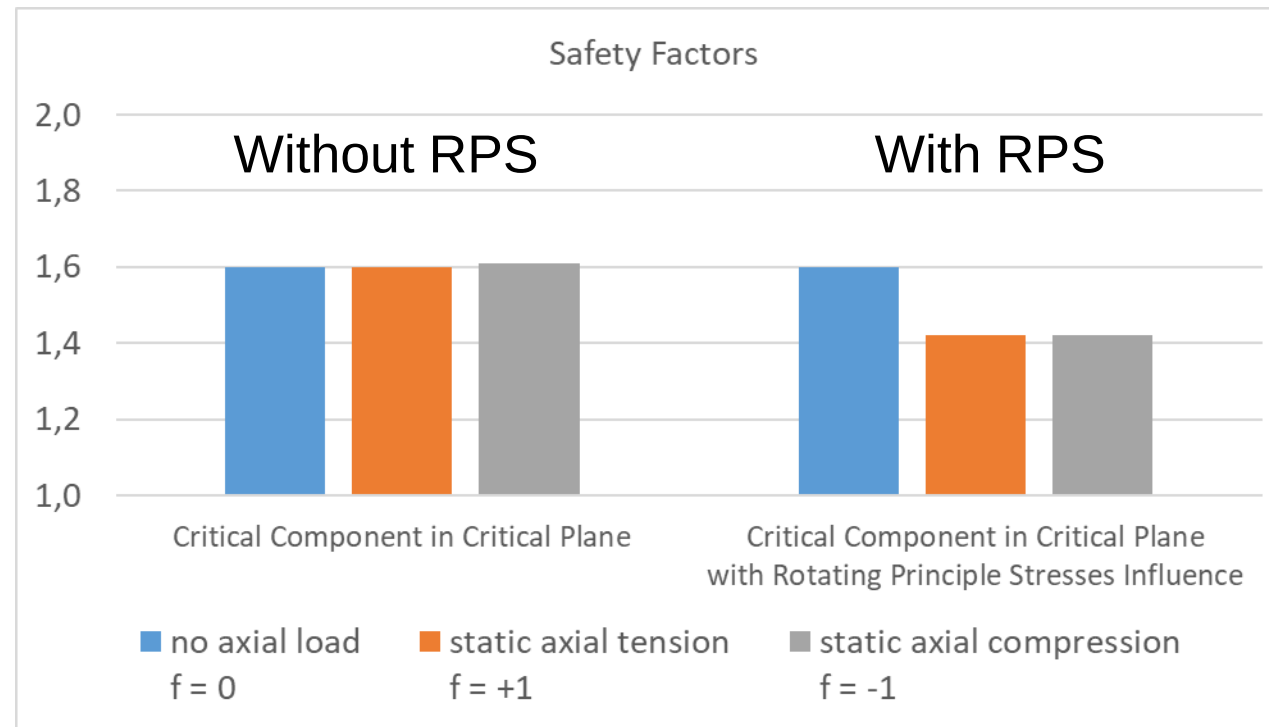
FE-model



$$\tau(t) = A \cdot (\sin(\omega t) + 1)$$

$$\sigma(t) = A \cdot f = \text{static axial load}$$

FEMFAT results



Steel (mat class 16)
In the example: $A = 0,4 \cdot \sigma_{zd}$

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Summary

FEMFAT documentation

$$RPS_{infl} = 1 - S_{NP} \cdot d_{NP}, \quad d_{NP} = \sqrt{I_6/I_5}$$

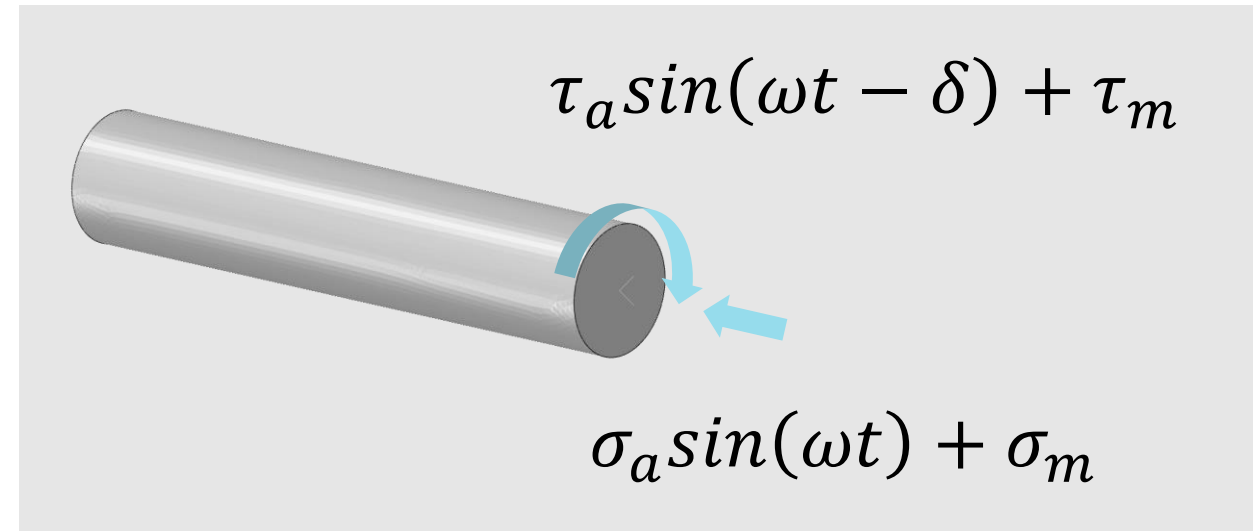
$$I = \sum_{k=1}^N \left(\sum_{i=1}^6 S_i^2(t_k) - \left(\sum_{i=1}^6 S_i(t_k) \cdot n_i \right)^2 \right)$$

Not easy to interpret

Project outcome

Analytical formula for the RPS influence
incl. validation + comparison with literature

$$RPS_{infl} = RPS_{infl}(\delta, \tau_m, \sigma_m, \dots)$$

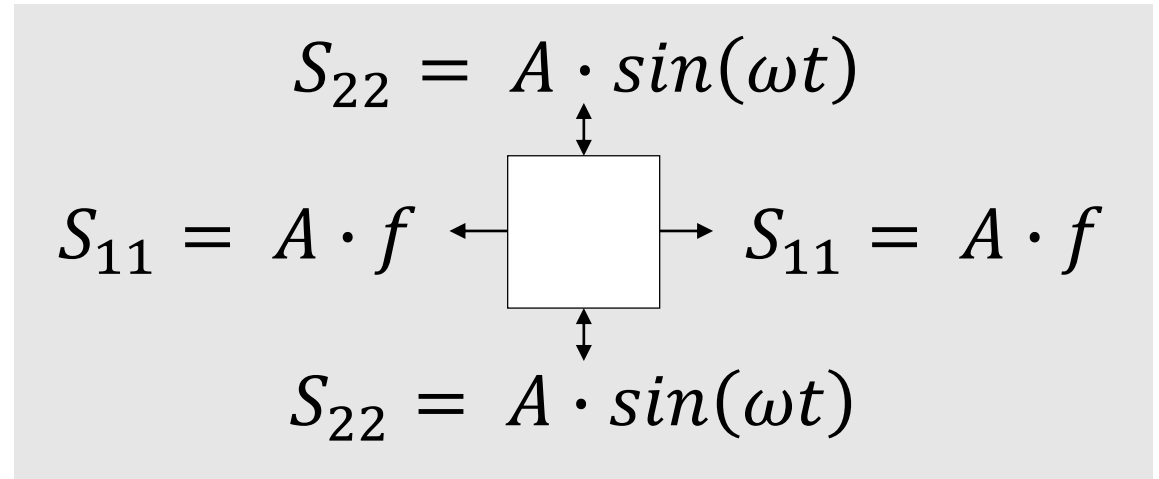


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First Analytical Case

Consider the stress history:

$$\sigma(t) = \begin{pmatrix} A \cdot f & 0 & 0 \\ 0 & A \cdot \sin(\omega t) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



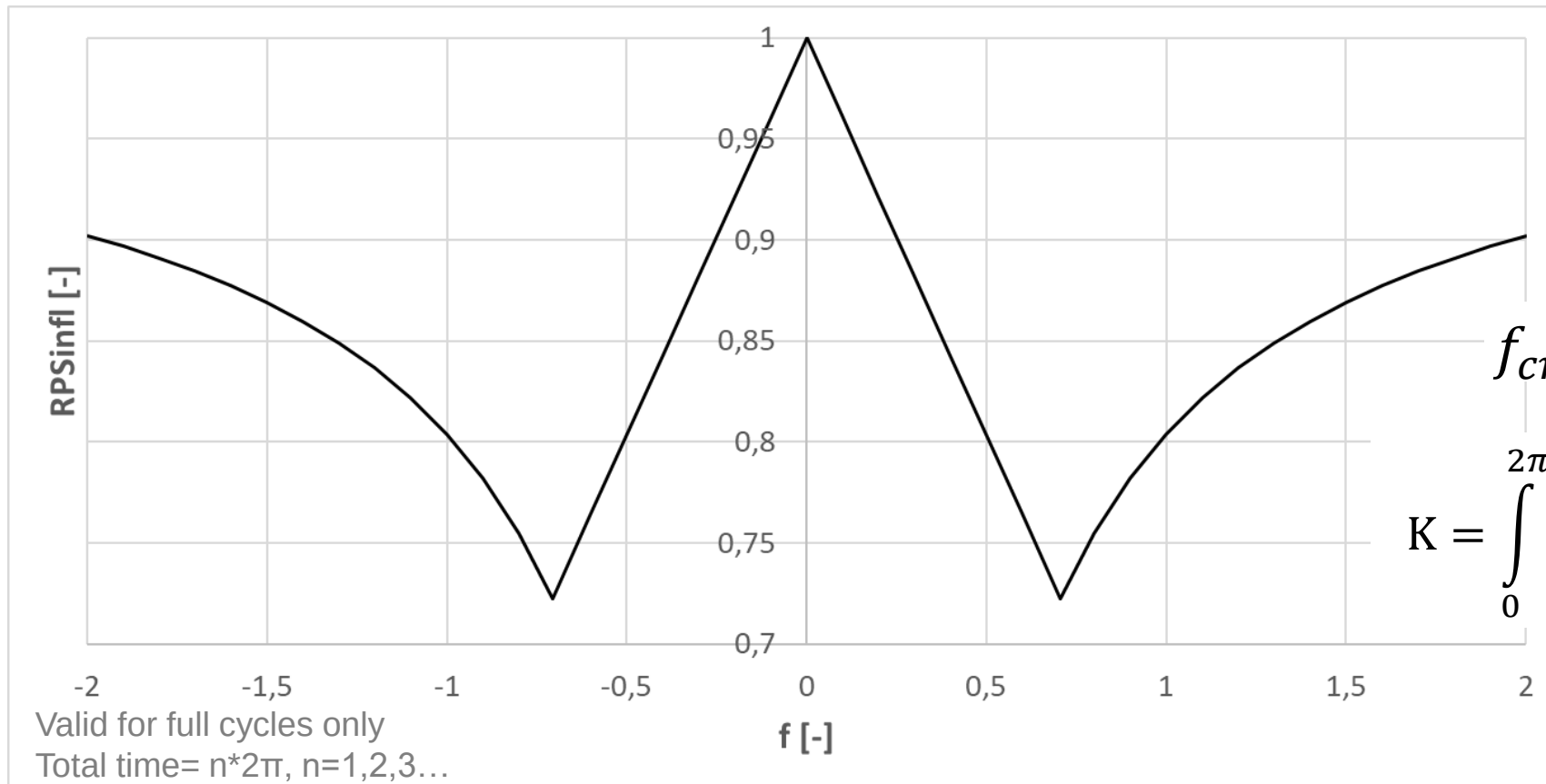
Integrating over one cycle and solving the eigenvalues:


$$RPS_{infl} = 1 - S_{NP} \cdot \sqrt{\frac{MIN(K; f^2)}{MAX(K; f^2)}}, \quad K = \int_0^{2\pi} \sin^2(\omega t) dt = \frac{1}{2}$$

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First Analytical Case – Example

$$\sigma(t) = \begin{pmatrix} A \cdot f & 0 & 0 \\ 0 & A \cdot \sin(\omega t) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$$f_{critical} = \pm\sqrt{K}$$

$$K = \int_0^{2\pi} \sin^2(\omega t) dt = \frac{1}{2}$$

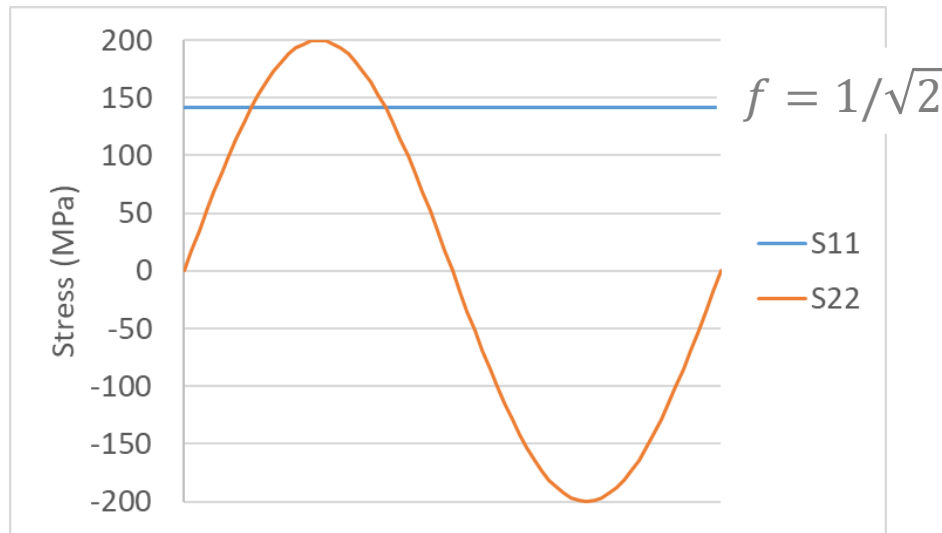
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First Analytical Case – Influence of Wave Shape

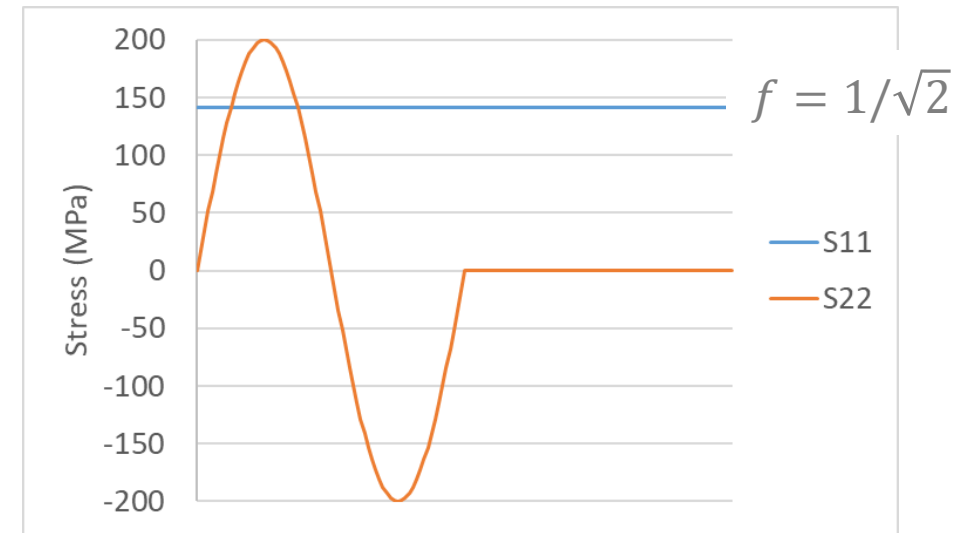
Applicable to any function $g(t)$ fulfilling:

$$\int_0^T g(t) dt = 0$$

$$\sigma(t) = \begin{pmatrix} A \cdot f & 0 & 0 \\ 0 & A \cdot g(t) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$$RPS_{infl} = 0.72$$



$$RPS_{infl} = 0.80$$

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Second Analytical Case

Two pendicular cyclic normal loads, with phase shift and mean stress

$$\sigma(t) = \begin{pmatrix} A \cdot f(\sin(\omega t) + B_1) & 0 & 0 \\ 0 & A(\sin(\omega t + \alpha) + B_2) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$$I_{6,5} = \frac{1}{2} A^2 \left((K + B_2^2 + f^2 K + f^2 B_1^2) \pm \sqrt{(K + B_2^2 + f^2 K + f^2 B_1^2)^2 - 4f^2 ((K + B_2^2)(K + B_1^2) - (K \cos(\alpha) + B_1 B_2)^2)} \right)$$

$$RPS_{infl} = 1 - S_{NP} \cdot d_{NP}, \quad d_{NP} = \sqrt{I_6 / I_5}$$

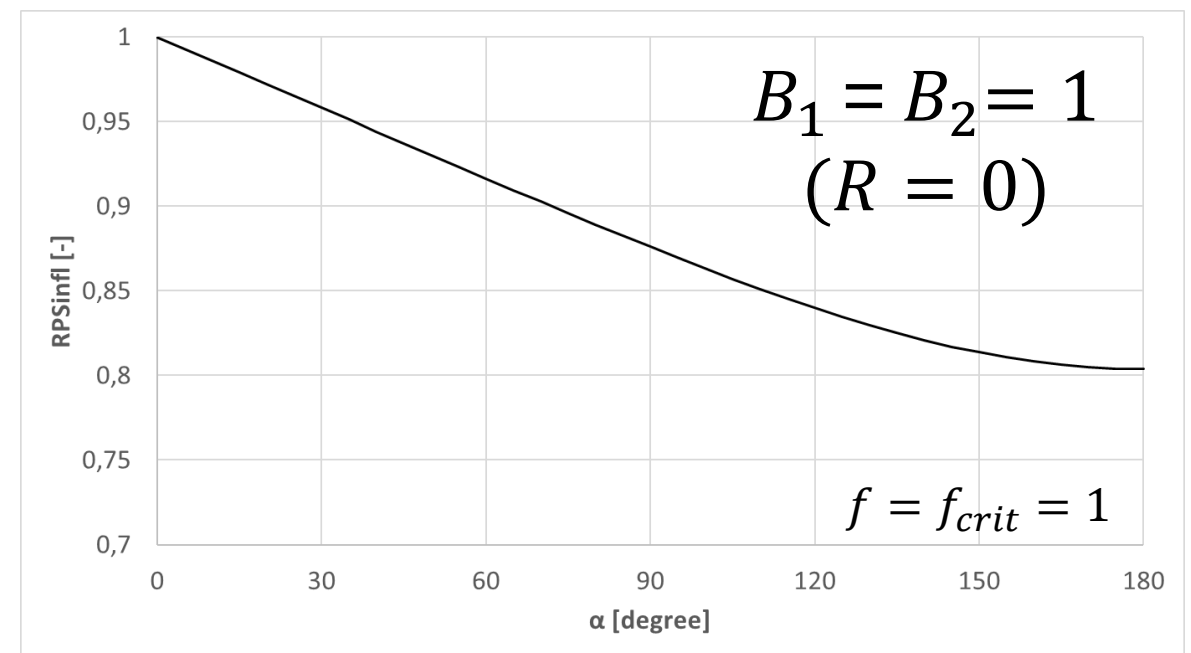
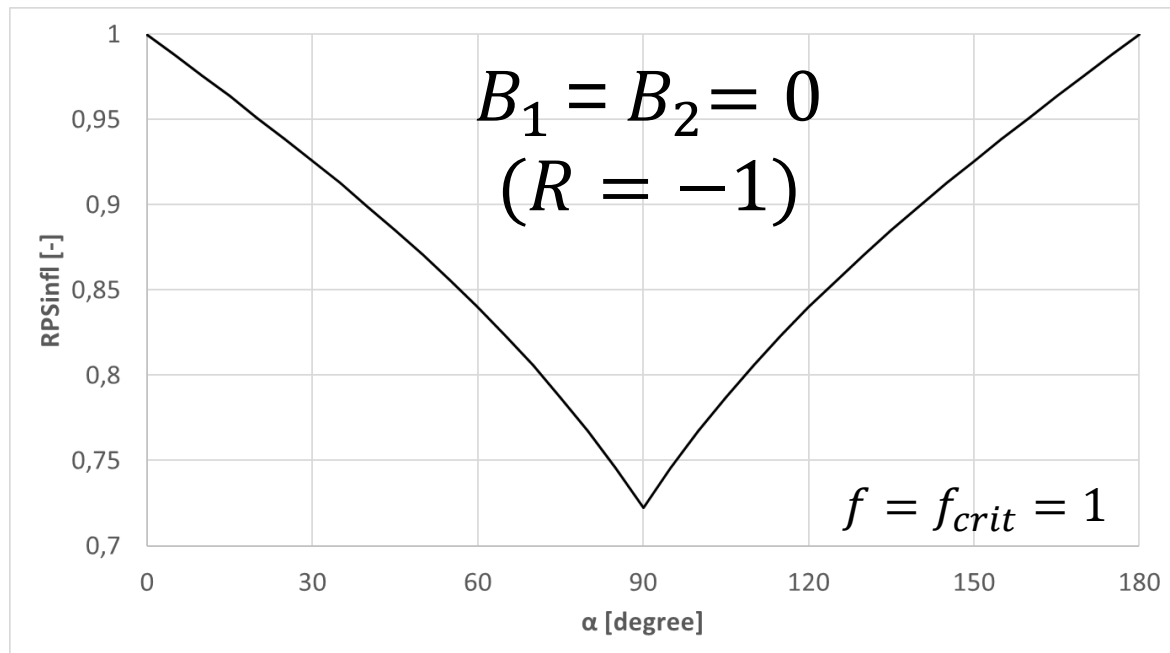
Most critical phase can be derived:

$$\alpha_{critical} = \begin{cases} \text{if } \text{abs}\left(\frac{-B_1 B_2}{K}\right) \leq 1: & \pm \arccos\left(\frac{-B_1 B_2}{K}\right) \\ \text{if } \frac{-B_1 B_2}{K} < -1: & \pm 180^\circ \\ \text{if } \frac{-B_1 B_2}{K} > 1: & 0^\circ \end{cases}$$

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Second Analytical Case – Examples

$$\sigma(t) = \begin{pmatrix} A \cdot f(\sin(\omega t) + B_1) & 0 & 0 \\ 0 & A(\sin(\omega t + \alpha) + B_2) & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

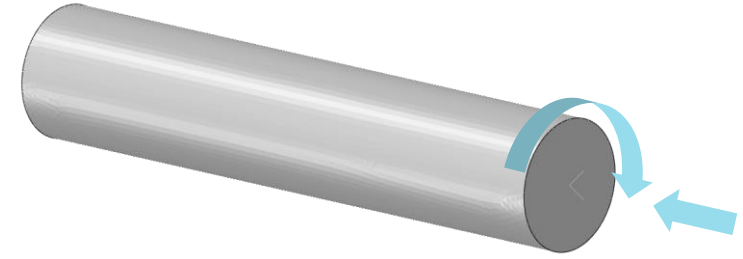


The critical phase is mean stress dependent

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Third Analytical Case

$$\sigma(t) = \begin{pmatrix} \sigma_a \sin(\omega t) + \sigma_m & \tau_a \sin(\omega t - \delta) + \tau_m & 0 \\ \tau_a \sin(\omega t - \delta) + \tau_m & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$



$$I_{6,5} = \frac{1}{2} \left((2\tau_a^2 K + 2\tau_m^2 + \sigma_a^2 K + \sigma_m^2) \pm \sqrt{(2\tau_a^2 K + 2\tau_m^2 + \sigma_a^2 K + \sigma_m^2)^2 - 8((\tau_a^2 K + \tau_m^2)(\sigma_a^2 K + \sigma_m^2) - (\sigma_a \tau_a K \cos(-\delta) + \sigma_m \tau_m)^2)} \right)$$

$$RPS_{infl} = 1 - S_{NP} \cdot d_{NP}, \quad d_{NP} = \sqrt{I_6 / I_5}$$

Validated using FEMFAT

The main challenge to validate the analytical solution:

- FEMFAT uses local FE-model stresses and not nominal stresses.

Validation method:

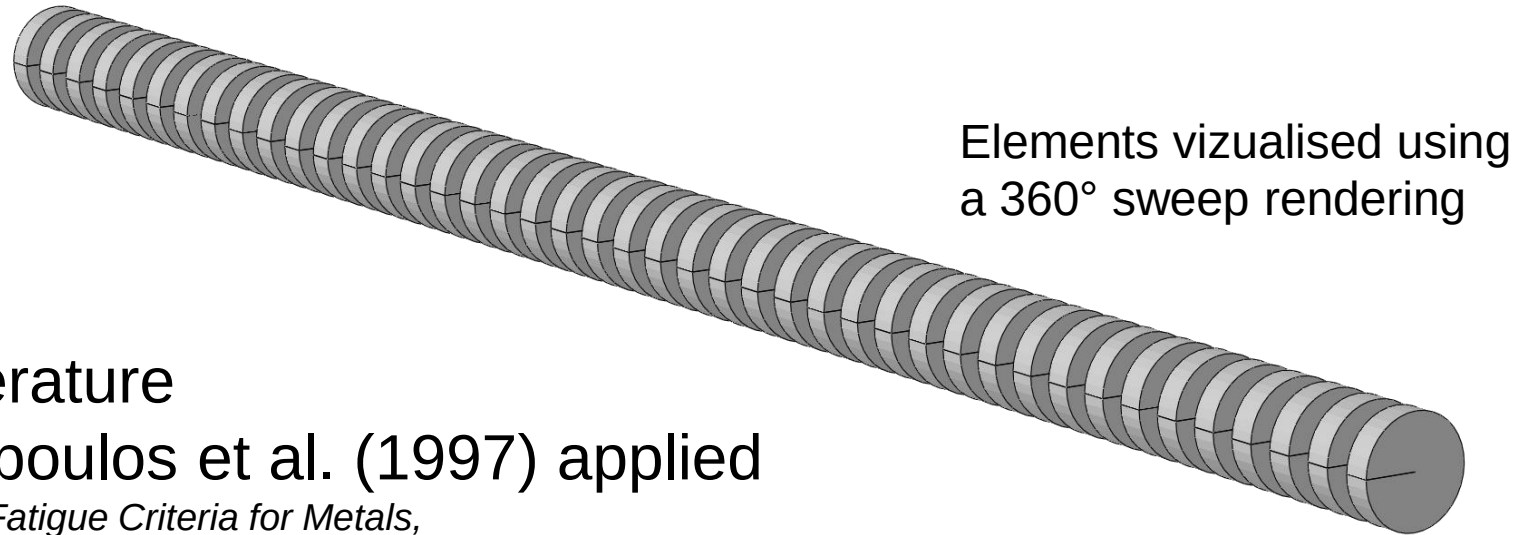
- Abaqus axi-symmetric elements with twist (CGAX4).

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Validation of Third Analytical Case

The successful solution to turn nominal stresses into local FE-stresses:

- FE-model of bars, using 2D elements CGAX4
- One element for each load case
- All elements loaded with unique normal and shear load histories



Elements visualised using
a 360° sweep rendering

Load cases chosen from literature

43 load cases from Papadopoulos et al. (1997) applied

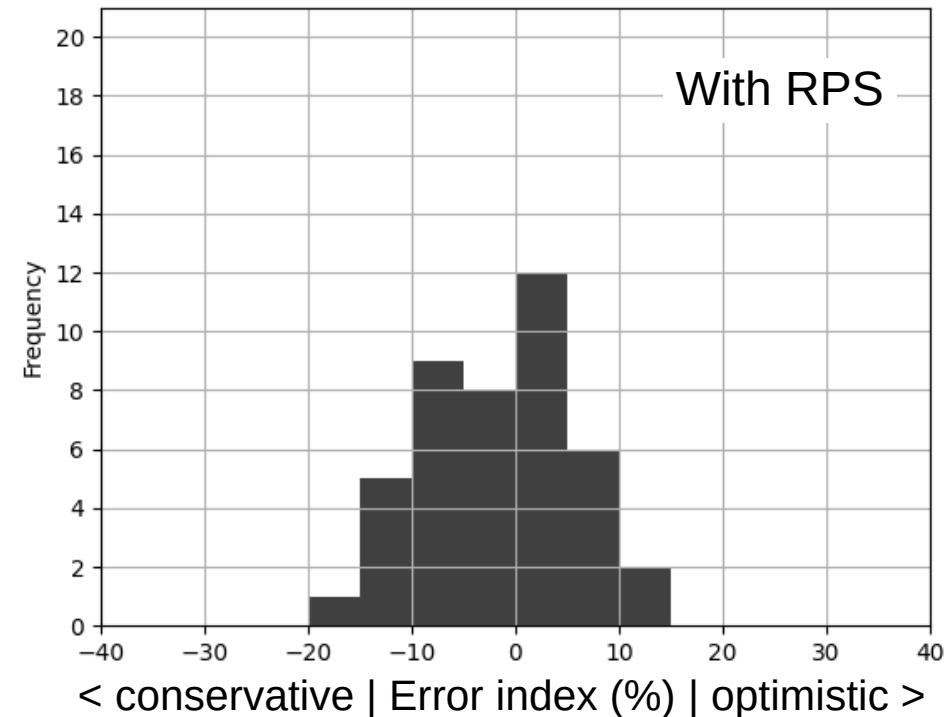
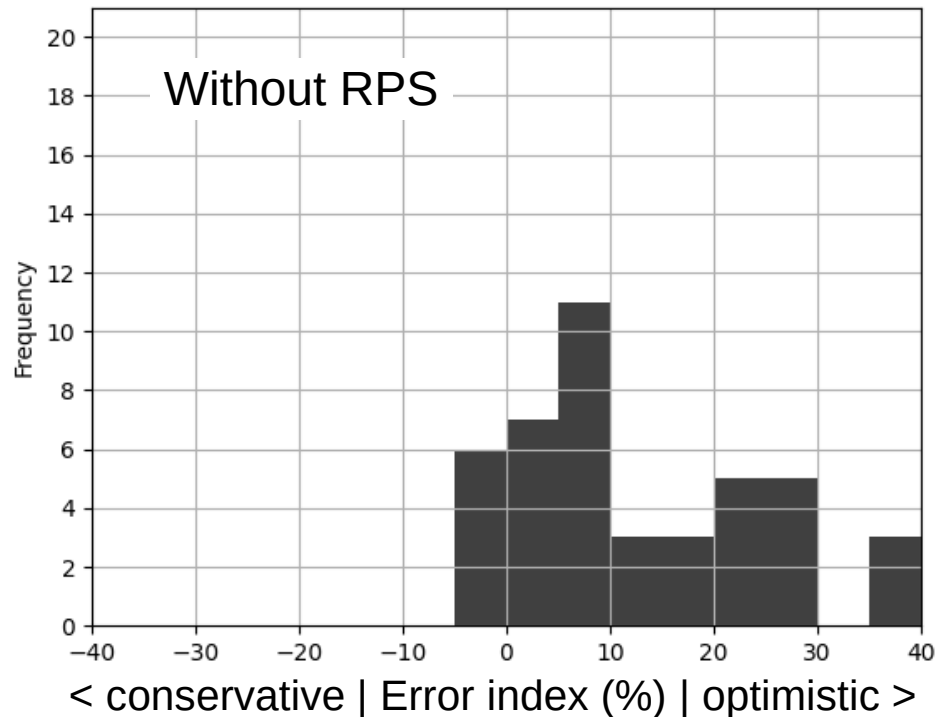
*“A Comparative Study of Multiaxial High-Cycle Fatigue Criteria for Metals,
Papadopoulos et al. Int. J. Fatigue Vol. 19, No. 3, pp. 219-235, 1997”*

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Comparison with Literature

FEMFAT results plotted in histogram

Material: Steel
(FEMFAT mat class 16)

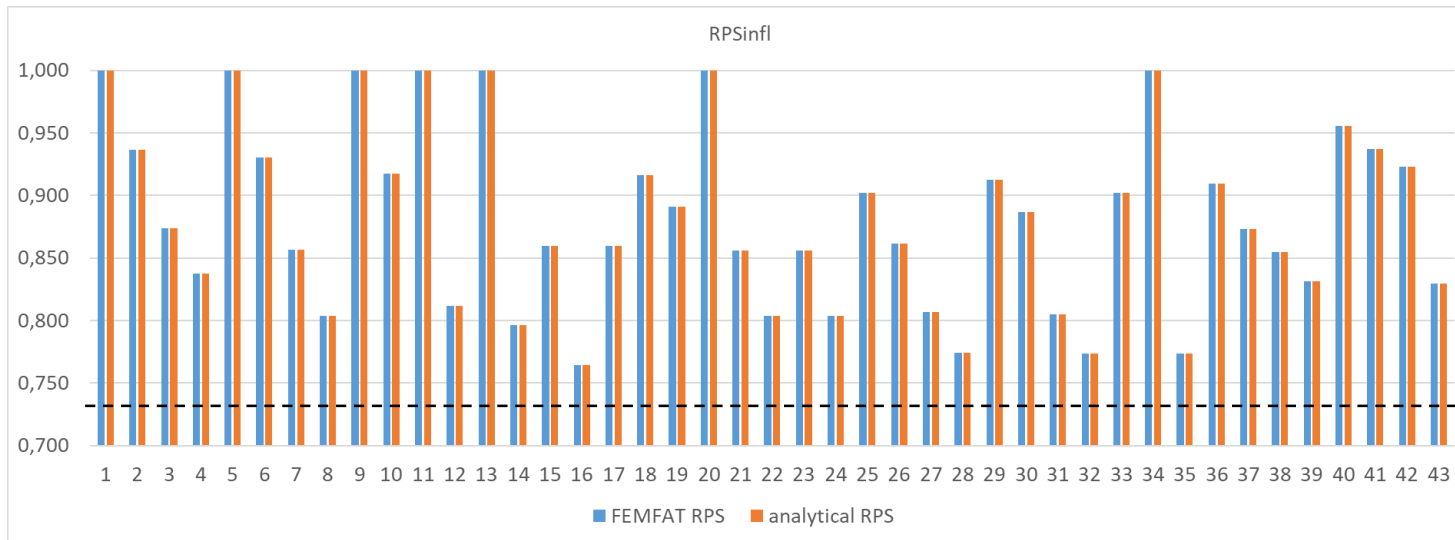


- Nominal stresses only, as in the referenced paper
- Sinusoidal wave form applied

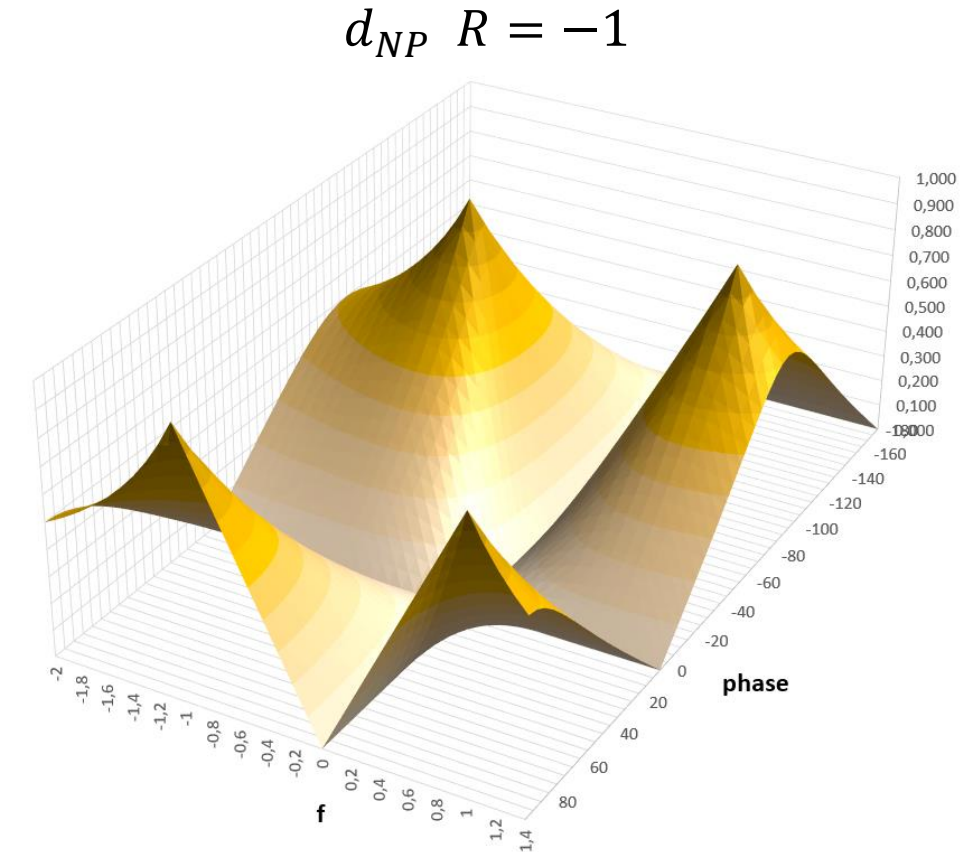
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Differentiability

None of the 43 load cases reached the maximum RPS penalty



- RPS is sensitive around local minima
- Non-differentiable points exist
- Due to the definition of d_{NP}

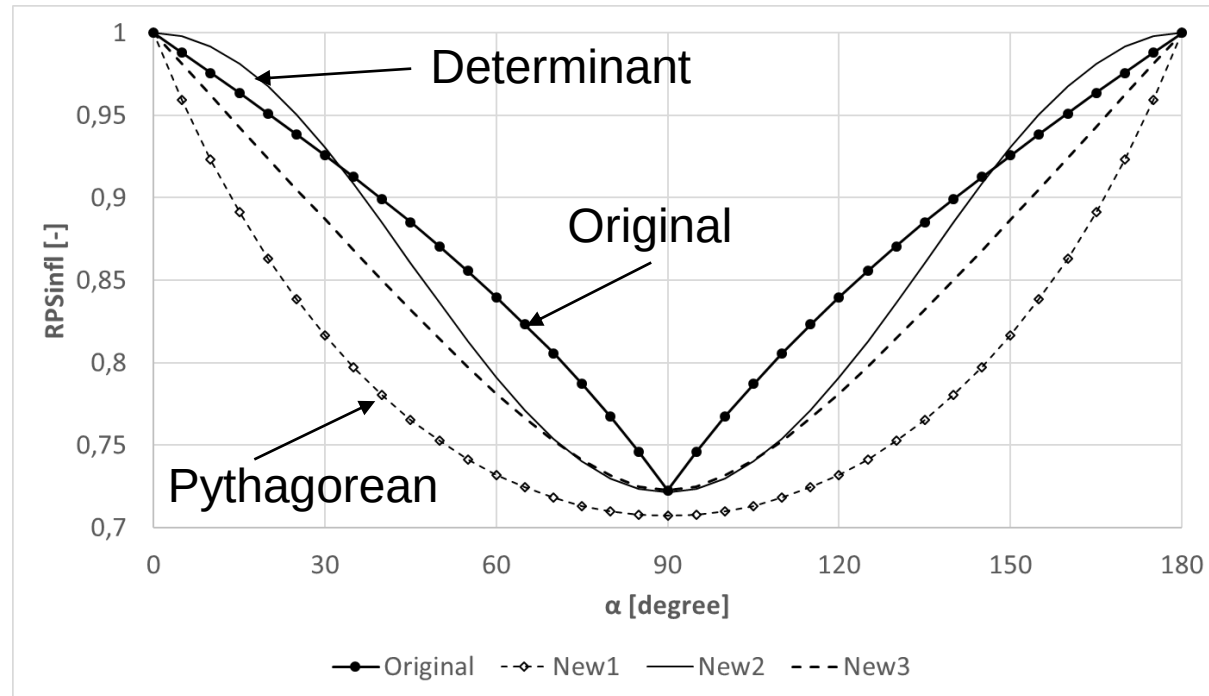


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Three Alternative Definitions

Proposed by MAN Energy Solutions:

- Reduces sensitivity around local minima
- Avoids non-differentiable minima



Original

$$RPS_{infl} = 1 - S_{NP} \cdot d_{NP}, \quad d_{NP} = \sqrt{I_6 / I_5}$$

Pythagorean

$$RPS_{infl,new,1} = \frac{\sqrt{I_5 + I_6}}{\sqrt{I_5} + \sqrt{I_6}}$$

Determinant based

$$RPS_{infl,new,2} = 1 - S_{NP} \cdot \frac{4I_5I_6}{(I_5 + I_6)^2}$$

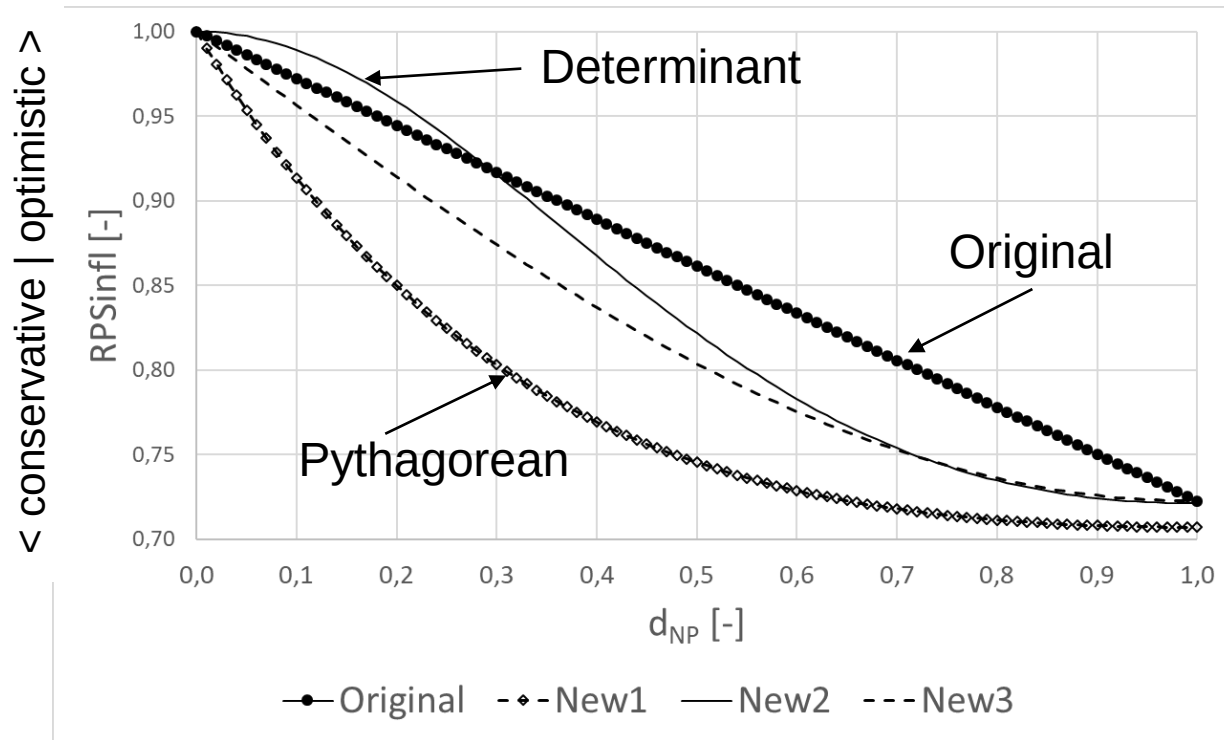
Sinusoidal

$$RPS_{infl,new,3} = 1 - S_{NP} \cdot \sin\left(d_{NP} \frac{\pi}{2}\right)$$

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Direct Mapping

The new formulations proposed by MAN-ES can be expressed as functions of d_{NP}



The Pythagorean definition is the most conservative

Original

$$RPS_{infl} = 1 - S_{NP} \cdot d_{NP}, \quad d_{NP} = \sqrt{I_6/I_5}$$

Pythagorean

$$RPS_{infl,new,1} = \frac{\sqrt{d_{NP}^2 + 1}}{d_{NP} + 1}$$

Determinant based

$$RPS_{infl,new,2} = 1 - S_{NP} \cdot \left(\frac{2d_{NP}}{1 + d_{NP}^2} \right)^2$$

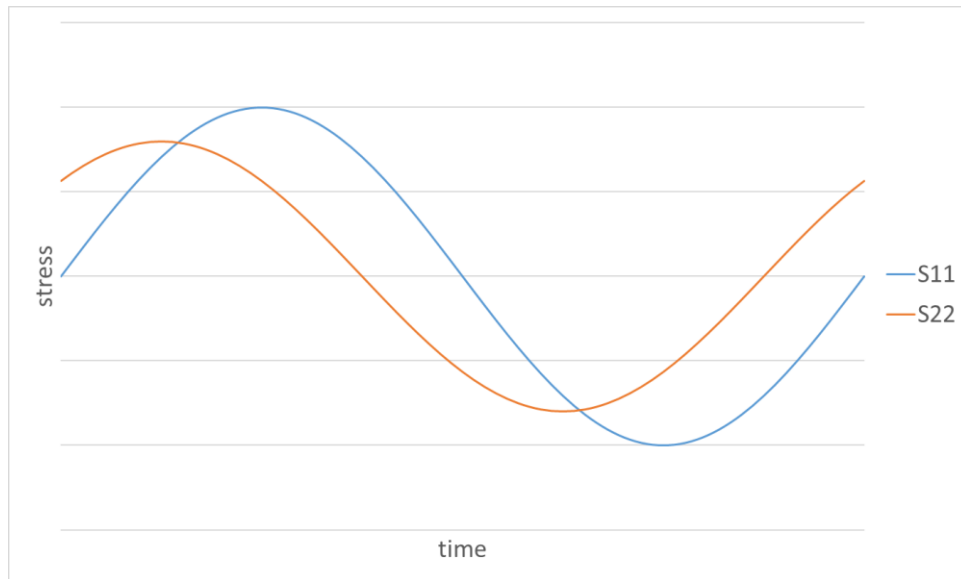
Sinusoidal

$$RPS_{infl,new,3} = 1 - S_{NP} \cdot \sin\left(d_{NP} \frac{\pi}{2}\right)$$

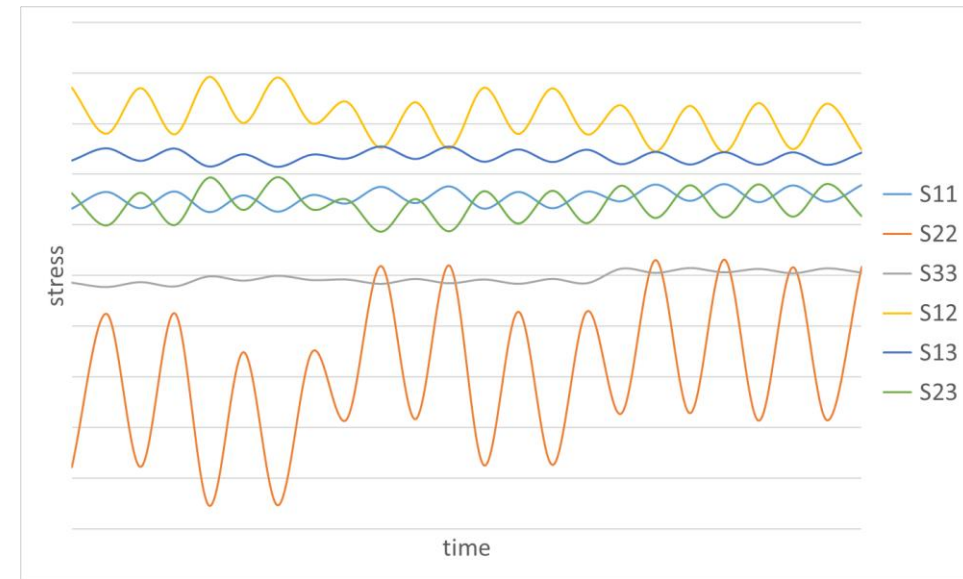
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In Practice

Load types studied



Actual service loads



Conservative approaches may be needed

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Summary

Analytical RPS formula

- Derived for special cases
- Critical phase and mean stress can be identified
- Validated using FEMFAT

Comparison with literature (steel)

- Correlation for steel is better with RPS influence than without

Changes in the definition, proposed by MAN-ES

- Sensitivity around local minima can be reduced
- More conservative results can be obtained

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Thank you for your attention

